LIBERTY PAPER SET

STD. 12 : Mathematics

Full Solution

Time : 3 Hours **ASSIGNTMENT PAPER 14**

PART A

1. (A) **2.** (D) **3.** (A) **4.** (B) **5.** (C) **6.** (D) **7.** (D) **8.** (B) **9.** (B) **10.** (A) **11.** (B) **12.** (A) **13.** (B) **14.** (C) **15.** (C) **16.** (C) **17.** (A) **18.** (B) **19.** (B) **20.** (A) **21.** (C) **22.** (A) **23.** (A) **24.** (A) **25.** (A) **26.** (D) **27.** (D) **28.** (A) **29.** (A) **30.** (B) **31.** (B) **32.** (A) **33.** (C) **34.** (C) **35.** (A) **36.** (D) **37.** (C) **38.** (A) **39.** (C) **40.** (B) **41.** (C) **42.** (B) **43.** (A) **44.** (A) **45.** (C) **46.** (C) **47.** (A) **48.** (A) **49.** (C) **50.** (B)

PART B

SECTION A

1.

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$$

$$= \tan^{-1}\left(\sqrt{\tan^2\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\left|\tan\frac{x}{2}\right|\right)$$

$$= \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$\left(\because\ 0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}\ \therefore\ \tan\frac{x}{2} > 0\right)$$

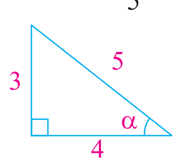
$$= \frac{x}{2}\quad \left(\because\ \frac{x}{2} \in \left(0, \frac{\pi}{2}\right) \subset \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$$

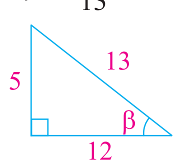
2.

L.H.S. $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$

$\cos^{-1}\frac{4}{5} = \alpha \quad , \quad \cos^{-1}\frac{12}{13} = \beta$

$\therefore \quad \cos\alpha = \frac{4}{5} \quad , \quad \cos\beta = \frac{12}{13}$





$\therefore \quad \sin\alpha = \frac{3}{5} \quad , \quad \sin\beta = \frac{5}{13}$

Here, $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \left(\frac{4}{5} \times \frac{12}{13}\right) - \left(\frac{3}{5} \times \frac{5}{13}\right)$$

$$= \frac{48}{65} - \frac{15}{65}$$

$$\cos(\alpha + \beta) = \frac{33}{65}$$

$$\therefore \quad \alpha + \beta = \cos^{-1}\left(\frac{33}{65}\right)$$

$$\therefore \quad \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$$

3.

$y = 5\cos x - 3\sin x$ Lkwt

Take differentiation by x both sides,

$$\frac{dy}{dx} = -5\sin x - 3\cos x$$

Now, take differentiation by x both sides,

$$\therefore \quad \frac{d^2y}{dx^2} = -5\cos x + 3\sin x$$

$$\therefore \quad \frac{d^2y}{dx^2} = -(5\cos x - 3\sin x)$$

$$\therefore \quad \frac{d^2y}{dx^2} = -y$$

$$\therefore \quad \frac{d^2y}{dx^2} + y = 0$$

4.

$$I = \int e^x \frac{(x-3)}{(x-1)^3}\, dx = \int e^x \left(\frac{(x-1)-2}{(x-1)^3}\right) dx$$

$$= \int e^x \left(\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right) dx$$

$$= \int e^x \left(\frac{1}{(x-1)^2} + \frac{d}{dx}\left(\frac{1}{(x-1)^2}\right)\right) dx$$

$$I = \frac{e^x}{(x-1)^2} + c \qquad \left(\because \int e^x (f(x) + f'(x))\, dx = e^x f(x) + c\right)$$

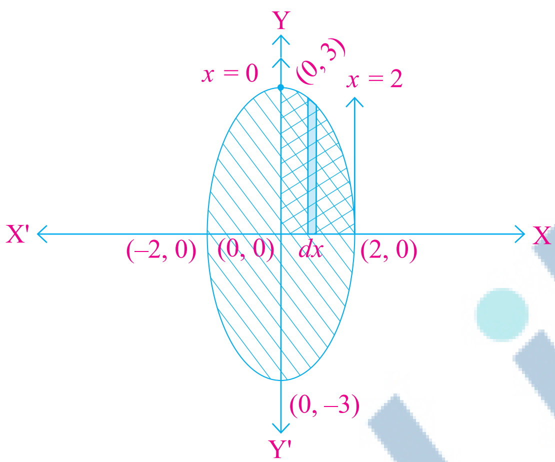
5.

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$a^2 = 4,\ a = 2$

$b^2 = 9,\ b = 3$

$b > a$



Required Area :

A = 4 × Area bounded on the first quadrant

$\therefore$ A = 4|I|

Now, $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$$\therefore\ y^2 = 9\left[1 - \frac{x^2}{4}\right]$$

$$= \frac{9}{4}(4 - x^2)$$

$$\therefore\ y = \frac{3}{2}\sqrt{4-x^2}$$

$$I = \int_0^2 y\ dx$$

$$I = \int_0^2 \frac{3}{2}\sqrt{4-x^2}\ dx$$

$$I = \frac{3}{2}\int_0^2 \sqrt{4-x^2}\ dx$$

$$= \frac{3}{2}\int_0^2 \sqrt{2^2-x^2}\ dx$$

$$I = \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$I = \frac{3}{2}\left[\left(\frac{2}{2}(0) + 2\sin^{-1}(1)\right) - (0)\right]$$

$$I = \frac{3}{2}\cdot 2 \cdot \frac{\pi}{2}$$

$$I = \frac{3\pi}{2}$$

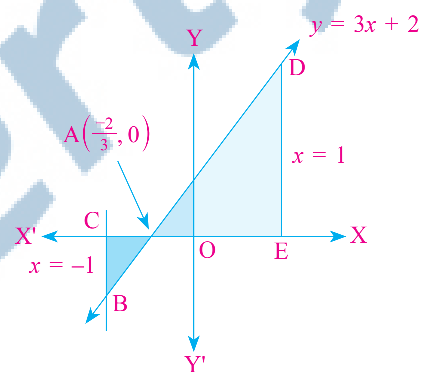
Now, A = 4|I|

$$= 4\left|\frac{3\pi}{2}\right|$$

$\therefore$ A = 6π sq. units

6.

As shown in the Fig, the line $y = 3x + 2$, meets X-axis at $\left(-\frac{2}{3}, 0\right)$ and its graph lies below X-axis for $x \in \left(-1, -\frac{2}{3}\right)$ and above X-axis for $x \in \left(-\frac{2}{3}, 1\right)$



The required area

= Area of the region ACBA

+ Area of the region ADEA

$$= \left|\int_{-1}^{-\frac{2}{3}} (3x+2)\, dx\right| + \int_{-\frac{2}{3}}^{1} (3x+2)\, dx$$

$$= \left|\left(\frac{3}{2}x^2 + 2x\right)_{-1}^{-\frac{2}{3}}\right| + \left(\frac{3}{2}x^2 + 2x\right)_{-\frac{2}{3}}^{1}$$

$$= \left|\left(\frac{3}{2}\left(\frac{4}{9}\right) + 2\left(\frac{-2}{3}\right)\right) - \left(\frac{3}{2}(1) + 2(-1)\right)\right|$$

$$+ \left(\frac{3}{2}(1) + 2(1)\right) - \left(\frac{3}{2}\left(\frac{4}{9}\right) + 2\left(-\frac{2}{3}\right)\right)$$

$$= \left|\frac{2}{3} - \frac{4}{3} - \frac{3}{2} + 2\right| + \frac{3}{2} + 2 - \frac{2}{3} + \frac{4}{3}$$

$$= \left|\frac{-1}{6}\right| + \frac{25}{6}$$

$$= \frac{1}{6} + \frac{25}{6}$$

$$= \frac{13}{3} \text{ sq. units}$$

7.

$$\cos\left(\frac{dy}{dx}\right) = a, \qquad \text{where, } a \in [-1, 1]$$

$\therefore \quad \frac{dy}{dx} = \cos^{-1}(a)$

$\therefore \quad dy = \cos^{-1}(a)\, dx$

→ Integrate both the sides,

$\therefore \int 1\, dy = \cos^{-1}(a) \int 1\, dx$

$\therefore \quad y = \cos^{-1}(a) \cdot x + c \qquad \ldots (1)$

$y = 2$ when $x = 0$

$\therefore\ 2 = \cos^{-1}(a) \cdot 0 + c$

$\therefore\ c = 2$

Put the value of c in equation (1),

$\therefore \quad y = \cos^{-1}(a) \cdot x + 2$

$\therefore\ \cos^{-1}(a) = \frac{y-2}{x}$

$\therefore\ a = \cos\left(\frac{y-2}{x}\right);$

which is required particular solution of given differential equation.

8.

Position vector of A $\vec{a} = \hat{i} + 2\hat{j} + 7\hat{k}$

Position vector of B $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

Position vector of C $\vec{c} = 3\hat{i} + 10\hat{j} - \hat{k}$

$\overrightarrow{AB} = \vec{b} - \vec{a}$

$= \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{BC} = \vec{c} - \vec{b}$

$= \hat{i} + 4\hat{j} - 4\hat{k}$

Now, $\overrightarrow{AB} = \lambda \overrightarrow{BC}$

$\therefore\ (\hat{i} + 4\hat{j} - 4\hat{k}) = \lambda\hat{i} + 4\lambda\hat{j} - 4\lambda\hat{k}$

$\therefore\ 1 = \lambda,\ 4 = 4\lambda,\ -4 = -4\lambda$

$\therefore\ \lambda = 1,\ \lambda = 1,\ \lambda = 1$

$\therefore$ Here, value of λ is equal.

$\therefore$ Therefore, A, B and C are collinear.

9.

$$L : \frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$$

$$\therefore \frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2}$$

$$L : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(-3\hat{i} + \frac{2p}{7}\hat{j} + 2\hat{k}\right)$$

$$\therefore \vec{b_1} = -3\hat{i} + \frac{2p}{7}\hat{j} + 2\hat{k}$$

Now, $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

$$\therefore \frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

$$M : \vec{r} = (\hat{i} + 5\hat{j} + 6\hat{k}) + \mu\left(\frac{-3p}{7}\hat{i} + \hat{j} - 5\hat{k}\right)$$

$$\therefore \vec{b_2} = \frac{-3p}{7} + \hat{j} - 5\hat{k}$$

→ L and M are perpendicular to each other;

$\vec{b_1} \cdot \vec{b_2} = 0$

$$\therefore \left(-3\hat{i} + \frac{2p}{7}\hat{j} + 2\hat{k}\right)\cdot\left(-\frac{3p}{7}\hat{i} + \hat{j} - 5\hat{k}\right) = 0$$

$$\therefore \frac{9p}{7} + \frac{2p}{7} - 10 = 0$$

$$\therefore \frac{11p}{7} = 10$$

$$\therefore p = \frac{70}{11}$$

10.

Suppose, A(1, –1, 2), B(3, 4, –2), C(0, 3, 2), D(3, 5, 6) are given points.

$\vec{b_1} = \overrightarrow{AB}$

= Position vector of B – Position vector of A

$= 2\hat{i} + 5\hat{j} - 4\hat{k}$

$\vec{b_2} = \overrightarrow{CD}$

= Position vector of D – Position vector of C

$= 3\hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{b_1} \cdot \vec{b_2} = (2\hat{i} + 5\hat{j} - 4\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 4\hat{k})$

$= 6 + 10 - 16$

$= 0$

$\therefore$ $\vec{b_1}$ and $\vec{b_2}$ are perpendicular to each other.

$\therefore$ Therefore, given lines are perpendicular to each other.

11.

P(B) = 0.5

P(A ∩ B) = 0.32

$$\therefore\ P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.32}{0.5}$$

$$= \frac{32}{100} \times \frac{10}{5}$$

$$= \frac{64}{100}$$

$$= 0.64$$

12.

Event E_1 : Person is men

Event E_2 : Person is women

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

Event A : Prob. that person is male given that person has grey hair

$$\therefore\ P(E_1 \mid A) = \frac{P(E_1) \cdot P(A \mid E_1)}{P(A)}$$

$$\therefore\ P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)$$

$P(A \mid E_1)$ = Men have grey hair

$$= \frac{5}{100}$$

$$= \frac{1}{20}$$

$P(A \mid E_2)$ = Women have grey hair

$$= \frac{0.25}{100}$$

$$= \frac{1}{400}$$

$$\therefore\ P(A) = \frac{1}{2} \times \frac{5}{100} + \frac{1}{2} \times \frac{0.25}{100}$$

$$= \frac{5 + 0.25}{200}$$

$$= \frac{5.25}{200}$$

$$\therefore\ P(E_1 \mid A) = \frac{\frac{1}{2} \times \frac{5}{100}}{\frac{5.25}{200}}$$

$$= \frac{5}{5.25}$$

$$= \frac{5 \times 100}{525}$$

$$= \frac{20}{21}$$

SECTION B

13.

Here $f : N \to N$, $f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

Take, $n_1 = 3$, $n_2 = 4$,

$f(n_1) = \dfrac{3+1}{2} = 2$ and $f(n_2) = f(4) = \dfrac{4}{2} = 2$

Here, $n_1 \neq n_2$ but $f(n_1) = f(n_2)$

$\therefore$ f is not one-one function.

Domain N = {1, 2, 3, 4, 5, 6,......}

$$f(n) = \begin{cases} \dfrac{n+1}{2} & \text{if } n \text{ is odd} \\ \dfrac{n}{2} & \text{if } n \text{ is even.} \end{cases}$$

$$f(1) = \frac{1+1}{2} = 1$$

$$f(2) = \frac{2}{2} = 1$$

$$f(3) = \frac{3+1}{2} = 2$$

$$f(4) = \frac{4}{2} = 2$$

$$f(5) = \frac{5+1}{2} = 3$$

$$f(6) = \frac{6}{2} = 3 \ .....$$

$\therefore$ R_f = {1, 2, 3, 4 ....} = N (co-domain)

$\therefore$ f is onto function.

14.

Here, A and B are both symmetric matrices,

$\therefore$ A' = A íkÚkk B' = B ... (1)

Now, X = AB – BA ÷uíkkt

X' = (AB – BA)'

= (AB)' – (BA)'

= B'A' – (A'B')

= BA – AB ($\because$ From equation (1))

= – (AB – BA)

= – X

$\therefore$ X is skew symmetric matrix,

$\therefore$ AB – BA is skew symmetric matrix.

15.

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{vmatrix}$$

$= 1[-\cos^2\alpha - \sin^2\alpha] - 0 + 0$

$= -1\,[\cos^2\alpha + \sin^2\alpha]$

$= -1 \neq 0$

$\therefore$ A^{-1} exists.

Co-factor of element 1 $\quad A_{11} = (-1)^2 \begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix}$

$= 1(-\cos^2\alpha - \sin^2\alpha)$

$= -1$

Co-factor of element 0 $\quad A_{12} = (-1)^3 \begin{vmatrix} 0 & \sin\alpha \\ 0 & -\cos\alpha \end{vmatrix}$

$= (-1)\ (0 - 0)$

$= 0$

Co-factor of element 0 $\quad A_{13} = (-1)^4 \begin{vmatrix} 0 & \cos\alpha \\ 0 & \sin\alpha \end{vmatrix}$

$= 1(0 - 0)$

$= 0$

Co-factor of element 0 $\quad A_{21} = (-1)^3 \begin{vmatrix} 0 & 0 \\ \sin\alpha & -\cos\alpha \end{vmatrix}$

$= (-1)\ (0 - 0)$

$= 0$

Co-factor of element $\cos\alpha$ $\ A_{22} = (-1)^4 \begin{vmatrix} 1 & 0 \\ 0 & -\cos\alpha \end{vmatrix}$

$= 1(-\cos\alpha - 0)$

$= -\cos\alpha$

Co-factor of element $\sin\alpha$ $\ A_{23} = (-1)^5 \begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix}$

$= (-1)(\sin\alpha - 0)$

$= -\sin\alpha$

Co-factor of element 0 $\quad A_{31} = (-1)^4 \begin{vmatrix} 0 & 0 \\ \cos\alpha & \sin\alpha \end{vmatrix}$

$= 1(0 - 0)$

$= 0$

Co-factor of element $\sin\alpha$ $\ A_{32} = (-1)^5 \begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix}$

$= (-1)(\sin\alpha - 0)$

$= -\sin\alpha$

Co-factor of element $-\cos\alpha$ $\ A_{33} = (-1)^6 \begin{vmatrix} 1 & 0 \\ 0 & \cos\alpha \end{vmatrix}$

$= 1(\cos\alpha - 0)$

$= \cos\alpha$

Here, $adj\ A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}\ adj\ A$

$$= \frac{1}{(-1)} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

16.

$y = \sin^{-1}x$ so, $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\therefore\ \sqrt{1-x^2}\cdot\frac{dy}{dx} = 1$

$\therefore\ \frac{d}{dx}\left(\sqrt{1-x^2}\cdot\frac{dy}{dx}\right) = 0$

$\therefore\ \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{d}{dx}\left(\sqrt{1-x^2}\right) = 0$

$\therefore\ \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} - \frac{dy}{dx}\cdot\frac{2x}{2\sqrt{1-x^2}} = 0$

$\therefore\ (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

Second Method :

$y = \sin^{-1}x$ is given,

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ i.e. $(1-x^2)\ y_1^2 = 1$

So, $(1-x^2)\cdot 2y_1y_2 + y_1^2(0-2x) = 0$

$\therefore\ (1-x^2)y_2 - xy_1 = 0$

17.

$f(x) = x^3 + \frac{1}{x^3}, \qquad x \neq 0$

$\therefore\ f'(x) = 3x^2 - \frac{3}{x^4}$

$= \frac{3x^6-3}{x^4}$

$= \frac{3(x^6-1)}{x^4}$

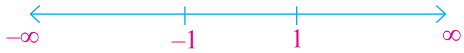
$\rightarrow$ For finding intervals,

$f'(x) = 0$

$\frac{3(x^6-1)}{x^4} = 0$

$\therefore\ x^6 - 1 = 0$

$\therefore\ x = \pm 1$



$\rightarrow \quad \forall\ x \in (-\infty, -1) \Rightarrow x < -1$

$\Rightarrow x^6 > 1$

$\Rightarrow x^6 - 1 > 0$

$\Rightarrow 3(x^6 - 1) > 0\ ,\ x^4 > 0$

$\Rightarrow \dfrac{3(x^6-1)}{x^4} > 0$

$\Rightarrow f'(x) > 0$

$\therefore$ f is strictly increasing function in the interval of $(-\infty, -1)$

$\rightarrow \quad \forall\ x \in (-1, 1) - \{0\} \Rightarrow -1 < x < 1$

$\Rightarrow 0 < x^6 < 1$

$\Rightarrow x^6 - 1 < 0$

$\Rightarrow 3(x^6 - 1) < 0,\ \ x^4 > 0$

$\Rightarrow \dfrac{3(x^6-1)}{x^4} < 0$

$\Rightarrow f'(x) < 0$

$\therefore$ f is strictly decreasing function in the interval of $(-1, 1) - \{0\}$.

$\rightarrow \quad \forall\ x \in (1, \infty) \quad \Rightarrow x > 1$

$\Rightarrow x^6 > 1$

$\Rightarrow x^6 - 1 > 0$

$\Rightarrow 3(x^6 - 1) > 0\ ,\ x^4 > 0$

$\Rightarrow \dfrac{3(x^6-1)}{x^4} > 0$

$\Rightarrow f'(x) > 0$

$\therefore$ f is strictly increasing function in the interval of $(1, \infty)$.

18.

Position vector of P

$\vec{p} = \hat{i} + 2\hat{j} - \hat{k}$

Position vector of Q

$\vec{q} = -\hat{i} + \hat{j} + \hat{k}$

(i) Point R divides line segment joining the points P and Q with ratio of 2 : 1 internally,

Position vector of point R $= \dfrac{\lambda\vec{q} + \vec{p}}{\lambda+1}$,

Here, $\lambda = 2$ ($\because$ internally)

$$= \frac{2(-\hat{i}+\hat{j}+\hat{k})+\hat{i}+2\hat{j}-\hat{k}}{2+1}$$

$$= \frac{-2\hat{i}+2\hat{j}+2\hat{k}+\hat{i}+2\hat{j}-\hat{k}}{3}$$

Position vector of point R $= \dfrac{-1}{3}\hat{i} + \dfrac{4}{3}\hat{j} + \dfrac{1}{3}\hat{k}$

(ii) Point R divides line segment joining the points P and Q with the ratio of 2 : 1 externally,

Position vector of point R $= \dfrac{-\lambda\vec{q} + \vec{p}}{-\lambda+1}$

Here, $\lambda = 2$ ($\because$ Externally)

$$= \frac{-2(-\hat{i}+\hat{j}+\hat{k})+\hat{i}+2\hat{j}-\hat{k}}{-2+1}$$

$$= \frac{2\hat{i}-2\hat{j}-2\hat{k}+\hat{i}+2\hat{j}-\hat{k}}{-1}$$

Position vector of point R $= -3\hat{i} + 3\hat{k}$

19.

L : $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$;

M : $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

$\therefore\ \vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$

$\therefore\ \vec{b_1} = \hat{i} - \hat{j} + \hat{k}$, and

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$;

$\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

Now, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$= -3\hat{i} + 0\hat{j} + 3\hat{k}$

$\neq \vec{0}$

$\therefore$ Lines are intersecting lines or skew lines.

$\vec{a_2} - \vec{a_1} = (2\hat{i} - \hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})$

$= \hat{i} - 3\hat{j} - 2\hat{k}$

Now, $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$

$= (\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (-3\hat{i} + 0\hat{j} + 3\hat{k})$

$= -3 + 0 - 6$

$= -9$

$\neq 0$

$\therefore$ Lines are skew line.

Shortest distance between two lines,

$$= \frac{\left|(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})\right|}{\vec{b_1}\times\vec{b_2}}$$

$$= \frac{|-9|}{\sqrt{9+0+9}}$$

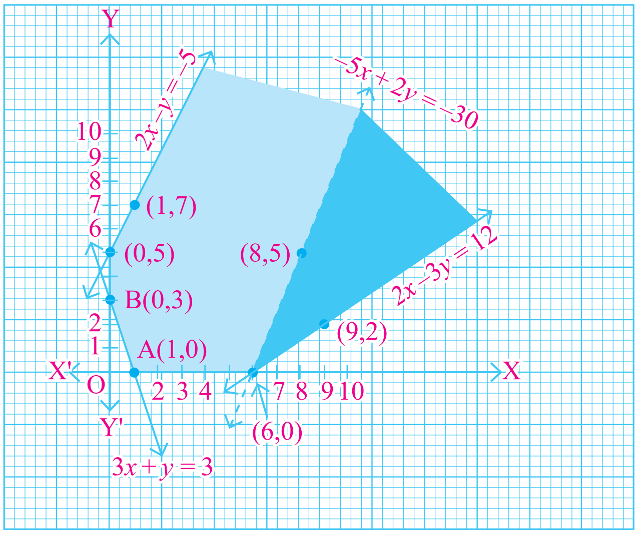
$$= \frac{9}{\sqrt{18}}$$

$$= \frac{9}{3\sqrt{2}}$$

$$= \frac{3}{\sqrt{2}} \text{ unit}$$

20.

First of all, let us graph the feasible region of the system of inequalities (2) to (5). The feasible region (shaded) is shown in the Fig 12.5. Observe that the feasible region is unbounded.



We now evaluate Z at the corner points.

Corner Point	$Z = -50x + 20y$
(0, 5)	100
(0, 3)	60
(1, 0)	–50
(6, 0)	–300 → Minimum

From this table, we find that –300 is the smallest value of Z at the corner point (6, 0). Can we say that minimum value of Z is – 300 ?

Note that if the region would have been bounded, this smallest value of Z is the minimum value of Z (Theorem 2). But here we see that the feasible region is unbounded. Therefore, – 300 may or may not be the minimum value of Z.

To decide this issue, we graph the inequality

$-50x + 20y < -300$ (see Step 3(ii) of corner Point Method.) i.e., $-5x + 2y < -30$ and check whether the resulting open half plane has points in common with feasible region or not. If it has common points, then –300 will not be the minimum value of Z.

Otherwise, –300 will be the minimum value of Z. As shown in the Fig., it has common points. Therefore, $Z = -50x + 20y$ has no minimum value subject to the given constraints.

In the above example, can you say whether $z = -50x + 20y$ has the maximum value 100 at (0, 5) ? For this, check whether the graph of $-50x + 20y > 100$ has points in common with the feasible region. (Why?)

21.

Event E_1 : coin is two hadded coin

Event E_2 : coin is biased

Event E_3 : coin is unbiased

$P(E_1) = \frac{1}{3}$,

$P(E_2) = \frac{1}{3}$,

$P(E_3) = \frac{1}{3}$

Event A : it shows head

$P(A \mid E_1) = 1$,

$P(A \mid E_2) = \frac{75}{100}$,

$P(A \mid E_3) = \frac{1}{2}$

The probability that it was the two headed coin,

$$\therefore\ P(E_1 \mid A) = \frac{P(E_1)\cdot P(A \mid E_1)}{P(A)}$$

$$P(A) = P(E_1)\cdot P(A \mid E_1) + P(E_2)\cdot P(A \mid E_2) + P(E_3)\cdot P(A \mid E_3)$$

$$= \frac{1}{3}\times 1 + \frac{1}{3}\times\frac{75}{100} + \frac{1}{3}\times\frac{1}{2}$$

$$= \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$$

$$= \frac{4+3+2}{12}$$

$$= \frac{9}{12}$$

$$= \frac{3}{4}$$

$$\therefore\ P(E_1 \mid A) = \frac{\frac{1}{3}\times 1}{\frac{3}{4}}$$

$$= \frac{4}{9}$$

SECTION C

22.

I is identity matrix of order 2,

$$\therefore\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Now, } I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \quad \text{......... (1)}$$

Now, $(I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right\}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\cdot\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\cdot\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

Now, $\cos\alpha + \sin\alpha\tan\frac{\alpha}{2}$

$$= \cos\alpha + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\cdot\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$$

$\left(\because \sin 2\theta = 2\sin\theta\cos\theta \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$$= \cos\alpha + 2\sin^2\frac{\alpha}{2}$$

$$= \cos\alpha + 1 - \cos\alpha$$

$$= 1$$

$-\sin\alpha + \cos\alpha\cdot\tan\frac{\alpha}{2}$

$$= -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \cos\alpha\cdot\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$$

$$= \sin\frac{\alpha}{2}\left[-2\cos\frac{\alpha}{2} + \frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right]$$

$$= \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\left[-2\cos^2\frac{\alpha}{2} + \cos\alpha\right]$$

$$= \tan\frac{\alpha}{2}\left[-2\cos^2\frac{\alpha}{2} + 2\cos^2\frac{\alpha}{2} - 1\right]$$

$$= -\tan\frac{\alpha}{2}$$

$$\therefore (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \quad ...(2)$$

From equation (1) and (2),

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

23.

$$A^{-1} = \begin{bmatrix} \frac{-14}{13} & \frac{-11}{13} & \frac{5}{13} \\ \frac{-11}{13} & \frac{-4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$|A^{-1}| = \frac{-14}{13}\left(\frac{-4}{169} - \frac{9}{169}\right) + \frac{+11}{13}\left(\frac{-11}{169} - \frac{15}{169}\right) + \frac{+5}{13}\left(\frac{-33}{169} + \frac{+20}{169}\right)$$

$$= \frac{-14}{13}\left(\frac{-13}{169}\right) + \frac{+11}{13}\left(\frac{-2}{169}\right) + \frac{+5}{13}\left(\frac{-13}{169}\right)$$

$$= \frac{14}{169}\ \frac{-22}{169}\ \frac{-5}{169}$$

$$= \frac{-13}{169}$$

$$= \frac{-1}{13} \neq 0$$

$\therefore$ $(A^{-1})^{-1}$ is exists.

$$(A^{-1})^{-1} = \frac{1}{|A^{-1}|}\ adj\ (A^{-1})$$

$$= -13\begin{bmatrix} \frac{-1}{13} & \frac{2}{13} & \frac{-1}{13} \\ \frac{2}{13} & \frac{-3}{13} & \frac{-1}{13} \\ \frac{-1}{13} & \frac{-1}{13} & \frac{-5}{13} \end{bmatrix} (\because \text{ from equation (2)})$$

$$(A^{-1})^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$\therefore (A^{-1})^{-1} = A$$

24.

$$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$$

Now, take differentiation by t both sides,

$$\therefore \frac{dx}{dt} = \frac{\sqrt{\cos 2t}\ \frac{d}{dt}(\sin^3 t) - (\sin^3 t)\frac{d}{dt}\sqrt{\cos 2t}}{(\sqrt{\cos 2t})^2}$$

$$= \frac{\sqrt{\cos 2t}\cdot 3\sin^2 t\cdot\cos t - \sin^3 t\cdot\frac{1}{2\sqrt{\cos 2t}}(-2\sin 2t)}{\cos 2t}$$

$$\therefore \frac{dx}{dt} = \frac{\cos 2t\cdot 3\sin^2 t\cdot\cos t + \sin^3 t\cdot\sin 2t}{(\cos 2t)^{\frac{3}{2}}} \quad ... (1)$$

Now, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ Lke

Differentiate by t both sides,

$$\frac{dy}{dt} = \frac{\sqrt{\cos 2t}\ \frac{d}{dt}(\cos^3 t) - \cos^3 t\ \frac{d}{dt}\sqrt{\cos 2t}}{(\sqrt{\cos 2t})^2}$$

$$= \frac{\sqrt{\cos 2t}\cdot 3\cos^2 t\cdot(-\sin t) - \cos^3 t\cdot\frac{1}{2\sqrt{\cos 2t}}(-2\sin 2t)}{\cos 2t}$$

$$\therefore \frac{dy}{dt} = \frac{-\cos 2t\cdot 3\cos^2 t\cdot\sin t + \cos^3 t\cdot\sin 2t}{(\cos 2t)^{\frac{3}{2}}} \quad ... (2)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\therefore \frac{dx}{dt} = \frac{\frac{-3\cos^2 t\cdot\sin t\cdot\cos 2t + \cos^3 t\cdot\sin 2t}{(\cos 2t)^{\frac{3}{2}}}}{\frac{3\sin^2 t\cdot\cos t\cdot\cos 2t + \sin^3 t\cdot\sin 2t}{(\cos 2t)^{\frac{3}{2}}}}$$

$$= \frac{-3\cos^2 t\cdot\sin t\cdot\cos 2t + \cos^3 t\cdot\sin 2t}{3\sin^2 t\cdot\cos t\cdot\cos 2t + \sin^3 t\cdot\sin 2t}$$

$$= \frac{\cos^2 t\,[-3\sin t \cdot \cos 2t + \cos t \cdot \sin 2t]}{\sin^2 t\,[3\cos t \cdot \cos 2t + \sin t \cdot \sin 2t]}$$

$$= \frac{\cos^2 t\,[-3\sin t\,(2\cos^2 t - 1) + \cos t \cdot \sin 2t]}{\sin^2 t\,[3\cos t\,(1 - 2\sin^2 t) + \sin t \cdot \sin 2t]}$$

$$= \frac{\cos^2 t\,[-6\sin t \cdot \cos^2 t + 3\sin t + \cos t \cdot \sin 2t]}{\sin^2 t\,[3\cos t - 6\cos t\,\sin^2 t + \sin t \cdot \sin 2t]}$$

$$= \frac{\cos^2 t\,[-6\sin t \cdot \cos^2 t + 3\sin t + \cos t \cdot 2\sin t \cdot \cos t]}{\sin^2 t\,[3\cos t - 6\cos t\,\sin^2 t + \sin t \cdot 2\sin t \cdot \cos t]}$$

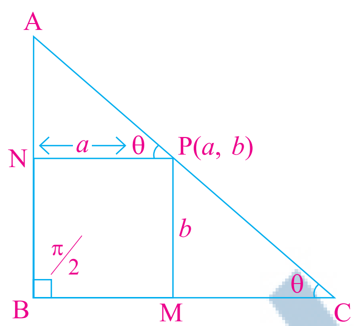
$$= \frac{\cos^2 t \cdot \sin t\,[-6\cos^2 t + 3 + 2\cos^2 t]}{\sin^2 t\,\cos t\,[3 - 6\sin^2 t + 2\sin^2 t]}$$

$$= \frac{\cos t\,[3 - 4\cos^2 t]}{\sin t\,[3 - 4\sin^2 t]} = \frac{3\cos t - 4\cos^3 t}{3\sin t - 4\sin^3 t}$$

$$= \frac{-\cos 3t}{\sin 3t}$$

$$\therefore \frac{dy}{dx} = -\cot 3t$$

25.



➪ $\overline{AC}$ is hypotenuse of right angle triangle ΔABC.

Point P(a, b) lies on the hypotenuse

Here, $\overline{PM} \perp \overline{BC}$ and $\overline{PN} \perp \overline{AB}$

Suppose, $\angle APN = \angle PCM = \theta$

ΔABC is right angle triangle $\theta \in \left(0, \frac{\pi}{2}\right)$

→ In ΔAPN,

$$\cos\theta = \frac{PN}{AP} = \frac{a}{AP}$$

$$\therefore \quad AP = \frac{a}{\cos\theta}$$

$$\therefore \quad AP = a\sec\theta$$

→ In ΔPMC,

$$\sin\theta = \frac{PM}{PC} = \frac{b}{PC}$$

$$\therefore \quad PC = \frac{b}{\sin\theta}$$

$$\therefore \quad PC = b\,\mathrm{cosec}\,\theta$$

Now, AC = AP + PC

$$\therefore \quad AC = a\sec\theta + b\,\mathrm{cosec}\,\theta \qquad \text{.......... (1)}$$

$$f(\theta) = a\sec\theta + b\,\mathrm{cosec}\,\theta$$

$$\therefore \quad f'(\theta) = a\sec\theta \cdot \tan\theta - b\,\mathrm{cosec}\,\theta \cdot \cot\theta$$

$$\therefore \quad f''(\theta) = a(\sec\theta \cdot \sec^2\theta + \tan\theta \cdot \sec\theta \cdot \tan\theta)$$
$$- b(\mathrm{cosec}\,\theta\,(-\mathrm{cosec}^2\theta) + \cot\theta(-\mathrm{cosec}\,\theta\,\cot\theta))$$

$$\therefore \quad f''(\theta) = a(\sec^3\theta + \sec\theta \cdot \tan^2\theta) + b(\mathrm{cosec}^3\theta + \mathrm{cosec}\,\theta\,\cot^2\theta)$$

$$\therefore \quad f''(\theta) > 0 \qquad \left(\because 0 < \theta < \frac{\pi}{2}\right)$$

→ For minimum length of hypotenuse,

$f'(\theta) = 0$

$$\therefore \quad a\sec\theta\tan\theta - b\,\mathrm{cosec}\,\theta\cot\theta = 0$$

$$\therefore \quad a\sec\theta\tan\theta = b\,\mathrm{cosec}\,\theta\cot\theta$$

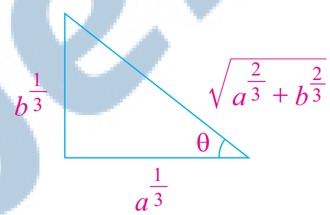
$$\therefore \quad \frac{a}{\cos\theta}\,\frac{\sin\theta}{\cos\theta} = \frac{b}{\sin\theta}\,\frac{\cos\theta}{\sin\theta}$$

$$\therefore \quad \frac{a}{\cos^3\theta} = \frac{b}{\sin^3\theta}$$

$$\therefore \quad \frac{\sin^3\theta}{\cos^3\theta} = \frac{b}{a}$$

$$\therefore \quad \tan^3\theta = \frac{b}{a}$$

$$\therefore \quad \tan\theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$$



$$\sec\theta = \frac{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}{a^{\frac{1}{3}}}$$

$$\mathrm{cosec}\,\theta = \frac{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}{b^{\frac{1}{3}}}$$

Put this value in equation (1),

$$\therefore AC = \frac{a\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{1}{2}}}{a^{\frac{1}{3}}} + \frac{b\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{1}{2}}}{b^{\frac{1}{3}}}$$

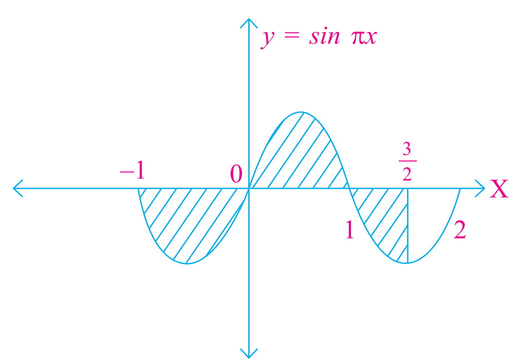
$$\therefore AC = \left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{1}{2}}\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{1}$$

$$\therefore AC = \left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

∴ Minimum length of hypotenuse $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

26.

➪



$\rightarrow \quad -1 < x < 0 \quad \Rightarrow sin\ \pi x < 0,\ x < 0$

$\Rightarrow x\ sin\ \pi x > 0$

$\Rightarrow |x\ sin\ \pi x| = x\ sin\ \pi x$

$\rightarrow \quad 0 < x < 1 \quad \Rightarrow sin\ \pi x > 0,\ x > 0$

$\Rightarrow x\ sin\ \pi x > 0$

$\Rightarrow |x\ sin\ \pi x| = x\ sin\ \pi x$

$\rightarrow \quad 1 < x < \frac{3}{2} \quad \Rightarrow sin\ \pi x < 0,\ x > 0$

$\Rightarrow x\ sin\ \pi x < 0$

$\Rightarrow |x\ sin\ \pi x| = -x\ sin\ \pi x$

Here, $f(x) = |x\ sin\ (\pi x)| = \begin{cases} x\ sin\ \pi\ x, & -1 \le x \le 1 \\ -x\ sin\ \pi\ x, & 1 \le x \le \frac{3}{2} \end{cases}$

So, $\int_{-1}^{\frac{3}{2}} |x\ sin\ (\pi x)|\ dx$

$$= \int_{-1}^{1} x\ sin\ \pi x\ dx + \int_{1}^{\frac{3}{2}} (-x\ sin\ dx)\ dx$$

$$= \int_{-1}^{1} x\ sin\ \pi x\ dx - \int_{1}^{\frac{3}{2}} (x\ sin\ \pi x)\ dx$$

$$= \left[\frac{-x\cos \pi\ x}{\pi} + \frac{\sin \pi\ x}{\pi^2}\right]_{-1}^{1} - \left[\frac{-x\cos \pi\ x}{\pi} + \frac{\sin \pi\ x}{\pi^2}\right]_{1}^{\frac{3}{2}}$$

(∵ ¾tzþ: Mktf÷Lk)

$$= \frac{2}{\pi} - \left[-\frac{1}{\pi^2} - \frac{1}{\pi}\right]$$

$$= \frac{3}{\pi} + \frac{1}{\pi^2}$$

27.

Method 1 :

The given differential equation can be written as :

$$\left[xy\ sin\left(\frac{y}{x}\right) - x^2\ cos\left(\frac{y}{x}\right)\right] dy = \left[xy\ cos\left(\frac{y}{x}\right) + y^2\ sin\left(\frac{y}{x}\right)\right] dx$$

$$\therefore \quad \frac{dy}{dx} = \frac{xy\ cos\left(\frac{y}{x}\right) + y^2\ sin\left(\frac{y}{x}\right)}{xy\ sin\left(\frac{y}{x}\right) - x^2\ cos\left(\frac{y}{x}\right)}$$

Dividing numerator and denominator on RHS by x^2, we get,

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{y}{x}\cos\left(\frac{y}{x}\right) + \left(\frac{y^2}{x^2}\right)\sin\left(\frac{y}{x}\right)}{\frac{y}{x}\sin\left(\frac{y}{x}\right) - \cos\left(\frac{y}{x}\right)} \quad \text{... (1)}$$

Clearly, equation (1) is a homogeneous differential equation of the form $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

$\rightarrow$ To solve it, we make the substitution

$$y = vx \quad \text{... (2)}$$

$$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{v\cos v + v^2 \sin v}{v \sin v - \cos v}$$

(using (1) and (2))

$$\therefore \quad x\frac{dv}{dx} = \frac{2v\cos v}{v\sin v - \cos v}$$

$$\therefore \quad \left(\frac{v\sin v - \cos v}{v\cos v}\right) dv = \frac{2dx}{x}$$

$$\int\left(\frac{v\sin v - \cos v}{v\cos v}\right) dv = 2\int \frac{1}{x}\,dx$$

$$\therefore \quad \int \tan v\ dv - \int \frac{1}{v}\,dv = 2\int \frac{1}{x}\,dx$$

$$\therefore \quad \log|\sec v| - \log|v| = 2\log|x| + \log|c_1|$$

$$\therefore \quad \log\left|\frac{\sec v}{vx^2}\right| = \log|c_1|$$

$$\therefore \quad \frac{\sec v}{vx^2} = \pm c_1$$

$\rightarrow$ Replacing v by $\frac{y}{x}$ in equation (3), we get,

$$\therefore \quad \frac{\sec\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)(x^2)} = c \qquad \text{where, } c = \pm c_1$$

$$\therefore \quad \sec\left(\frac{y}{x}\right) = cxy$$

which is the general solution of the given differential equation.

Method 2 :

$$\left(\frac{x\ dy - y\ dx}{x^2}\right) y\ sin\ \frac{y}{x} = \left(\frac{y\ dx + x\ dy}{x}\right) cos\ \frac{y}{x}$$

$$\therefore \quad d\left(\frac{y}{x}\right) sin\ \frac{y}{x} = \frac{d(xy)}{xy}\ cos\ \frac{y}{x}$$

$$\therefore \quad d\left(\frac{y}{x}\right) tan\ \frac{y}{x} = \frac{d(xy)}{xy}$$

$$\therefore \quad \log\left|\sec\frac{y}{x}\right| = \log|cxy|$$

$$\therefore \quad \sec\frac{y}{x} = cxy$$